Statistics

Lecture 6

Feb 19-8:47 AM

Class Quiz 5

Given $P(A) = .002$

1) Find $P(A)$ in reduced fraction.

$.002$ $\boxed{\text{MATH}}$ $\boxed{1{:}\blacktriangleright\text{Frac}}$ $\boxed{\text{Enter}}$ $\dfrac{1}{500}$

2) Find $P(\bar{A})$ in percentage.

$P(\bar{A}) = 1 - P(A) = 1 - .002 = .998 = \boxed{99.8\%}$

3) Find $\dfrac{P(A)}{P(\bar{A})}$ in reduced fraction.

$\dfrac{.002}{.998} = \boxed{\dfrac{1}{499}}$

$.002$ $\boxed{\div}$ $.998$ $\boxed{\text{Math}}$ $\boxed{1{:}\blacktriangleright\text{Frac}}$ $\boxed{\text{Enter}}$

Apr 4-7:58 AM

SG 11

Addition Rule

Keyword OR $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Single Action event

ex: $P(A) = .4$, $P(B) = .7$, $P(A \text{ and } B) = .25$

1) $P(\overline{A}) = 1 - P(A) = \boxed{.6}$

2) $P(\overline{B}) = 1 - .7 = \boxed{.3}$

3) $P(\overline{A \text{ and } B}) = 1 - P(A \text{ and } B) = 1 - .25 = \boxed{.75}$

4) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
$= .4 + .7 - .25 = \boxed{.85}$

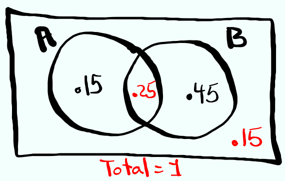
Addition Rule

5) $P(\overline{A \text{ or } B}) = 1 - P(A \text{ or } B) = 1 - .85 = \boxed{.15}$

6) Make Venn Diagram.

$.4 - .25 = .15$

$.7 - .25 = .45$



Total = 1

Apr 4-8:14 AM

$P(HB) = .6$

$P(FF) = .3$

$P(HB \text{ and } FF) = .25$

1) $P(\overline{HB}) = 1 - .6 = \boxed{.4}$

2) $P(\overline{FF}) = 1 - .3 = \boxed{.7}$

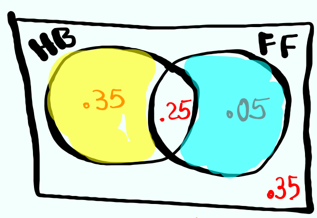
3) $P(HB \text{ or } FF) = P(HB) + P(FF) - P(HB \text{ and } FF)$
$= .6 + .3 - .25 = \boxed{.65}$

Addition Rule

4) Make Venn Diagram.

$.6 - .25 = .35$

$.3 - .25 = .05$



Total = 1

5) $P(HB \text{ only}) = .35$

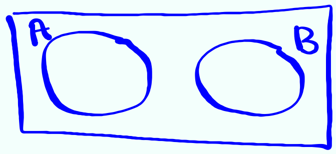
6) $P(FF \text{ only}) = .05$

7) $P(\text{order one, not both}) = .35 + .05 = \boxed{.4}$

Apr 4-8:25 AM

Mutually Exclusive Events } No overlap: A & B cannot happen together

Disjoint Events



$P(A \text{ and } B) = 0$

$P(A) = .15$ $\quad$ $P(B) = .75$ $\quad$ A & B are M.E.E.

1) $P(\bar{A}) = 1 - .15 = \boxed{.85}$

2) $P(\bar{B}) = 1 - .75 = \boxed{.25}$

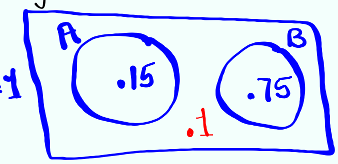
3) $P(A \text{ and } B) = \boxed{0}$

4) $P(\overline{A \text{ and } B}) = 1 - P(A \text{ and } B) = 1 - 0 = \boxed{1}$

5) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
$= .15 + .75 - 0 = \boxed{.9}$

6) Construct Venn Diagram

Total = 1



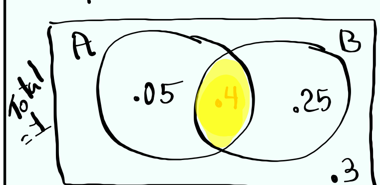
Apr 4-8:35 AM

De Morgan's Law:

$P(\bar{A} \text{ and } \bar{B}) = P(\overline{A \text{ or } B})$

$P(\bar{A} \text{ or } \bar{B}) = P(\overline{A \text{ and } B})$

Complete the Venn Diagram below



1) $P(A) = .45$

2) $P(A \text{ only}) = .05$

3) $P(B) = .65$

4) $P(B \text{ only}) = .25$

5) $P(A \text{ or } B) = .45 + .65 - .4 = \boxed{.7}$

6) $P(\bar{A} \text{ and } \bar{B}) = P(\overline{A \text{ or } B}) = 1 - .7 = \boxed{.3}$

De Morgan's Law

7) $P(\bar{A} \text{ or } \bar{B}) = P(\overline{A \text{ and } B}) = 1 - .4 = \boxed{.6}$

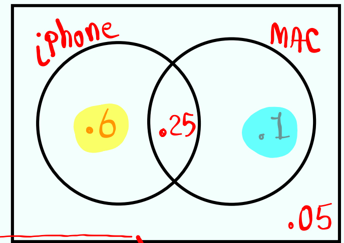
Apr 4-8:46 AM

$P(\text{iPhone}) = .85$

$P(\text{MAC}) = .35$

$P(\text{iphone and MAC}) = .25$

1) Construct Venn Diagram



$P(\overline{\text{iphone}} \text{ and } \overline{\text{MAC}}) = P(\overline{\text{iphone OR Mac}})$ Total = 1

De Morgan's Law

$= 1 - .95 = \boxed{.05}$

$P(\overline{\text{iphone}} \text{ OR } \overline{\text{MAC}}) = P(\overline{\text{iphone and MAC}}) = 1 - .25$

$= \boxed{.75}$

P(iphone only OR MAC only, not both)

$= .6 + .1 = \boxed{.7}$

SG 11

Apr 4-8:57 AM

SG 12

Odds in favor of event E

Notation $a : b$

# of times E happens

# of times $\overline{E}$ happens

Assume odds in favor of event E are

$3 : 17$

E happens 3 times

$\overline{E}$ happens 17 times

20 total times

odds against event E → $b : a$

$17 : 3$

Apr 4-9:21 AM

Consider a full deck of playing cards

52 Cards 4 Aces

1) P(Draw an Ace)

$\frac{4}{52} = \boxed{\frac{1}{13}}$

2) odds in favor of drawing an Ace

Aces : # $\overline{\text{Aces}}$

4 : 48 ⇒ $\boxed{1:12}$

4 ÷ 48 MATH 1:▶Frac Enter $\frac{1}{12}$

3) odds against drawing an Ace

Favor an Ace 1:12 $\boxed{12:1}$

Apr 4-9:25 AM

20 students 8 Females 12 Males

1) P(Select a female) = $\frac{8}{20} = \boxed{\frac{2}{5}}$

2) odds in favor of selecting a female.

8 females : 12 $\overline{\text{females}}$ ⇒ 2:3

3) odds against selecting a female.

3:2

Apr 4-9:30 AM

If odds in favor of event E are $a:b$,

then $P(E)=\dfrac{a}{a+b}$, $P(\bar{E})=\dfrac{b}{a+b}$.

Ex: odds in favor of event E are $3:47$.

1) Find odds against event E.

$\boxed{47:3}$

2) Find $P(E)=\dfrac{3}{3+47}=\boxed{\dfrac{3}{50}}=\boxed{.06}$

3) Find $P(\bar{E})=\dfrac{47}{3+47}=\boxed{\dfrac{47}{50}}=\boxed{.94}$

Apr 4-9:34 AM

If $P(E)$ is given, then

odds in favor of event E are

$P(E):P(\bar{E})$

Always simplify.

Suppose $P(E)=.002$

1) $P(\bar{E})=1-P(E)=\boxed{.998}$

2) odds in favor of event E.

$P(E):P(\bar{E})$

$.002:.998 \Rightarrow \boxed{1:499}$

$.002$ $\boxed{\div}$ $.998$ $\boxed{\text{MATH}}$ $\boxed{\text{1:}\blacktriangleright\text{Frac}}$ $\boxed{\text{Enter}}$ $\dfrac{1}{499}$

3) odds against event E.

$\boxed{499:1}$

Apr 4-9:38 AM

Suppose $P(\text{Dodgers win W.S.}) = .85$

$P(W) = .85$

1) $P(\overline{W}) = 1 - .85 = \boxed{.15}$

2) odds in $\boxed{\text{favor}}$ of Dodgers winning the World Series.

$P(W) : P(\overline{W})$

$.85 : .15$

$\rightarrow \boxed{17 : 3}$

3) odds against? $\boxed{3 : 17}$

Apr 4-9:44 AM

Thinking of placing a bet:

\# bet : \# Net Profit

$17 : $3
on Dodgers to win   Net Profit

+120
-175

$3 : $17
on Dodgers not to win   Net Profit

Apr 4-9:47 AM

Multiplication Rule

Keyword AND $P(A \text{ and } B)$

multiple Action event — A happens then B happens

Independent Events

one outcome does not change the prob. of next outcome.

$P(\text{Boy}) = .5$ Fair Coin

$P(\text{Girl}) = .5$ $P(\text{Tails}) = .5$

multiple-choice exam

each question has 4 choices but only one Correct choice

$P(\text{Correct}) = \frac{1}{4}$ $P(\overline{\text{Correct}}) = \frac{3}{4}$

Apr 4-9:51 AM

If A and B are independent events, then

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

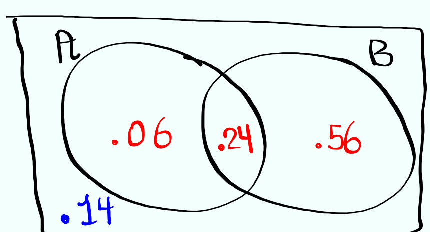
Given $P(A) = .3$, $P(B) = .8$

A and B are independent events

1) $P(\overline{A}) = \boxed{.7}$ 2) $P(\overline{B}) = \boxed{.2}$

3) $P(A \text{ and } B) = P(A) \cdot P(B) = (.3)(.8) = \boxed{.24}$

4) Venn Diagram



Total = 1

Apr 4-9:57 AM

Suppose a loaded coin is tossed twice.

$P(T) = .3 \qquad P(H) = .7$

Sample Space — Complete list of all possible outcomes

T T	$P(TT) = (.3)(.3) = \boxed{.09}$
T H	$P(TH) = (.3)(.7) = \boxed{.21}$
H T	$P(HT) = (.7)(.3) = \boxed{.21}$
H H	$P(HH) = (.7)(.7) = \boxed{.49}$

Total Prob. = 1

Apr 4-10:03 AM

Consider a full deck of playing cards
52 Cards, 4 Aces

Draw 2 Cards with replacement

Sample Space

A A	$P(AA) = \frac{4}{52} \cdot \frac{4}{52} = \boxed{\frac{1}{169}}$
A $\bar{A}$	$P(A\bar{A}) = \frac{4}{52} \cdot \frac{48}{52} = \boxed{\frac{12}{169}}$
$\bar{A}$ A	$P(\bar{A}A) = \frac{48}{52} \cdot \frac{4}{52} = \boxed{\frac{12}{169}}$
$\bar{A}$ $\bar{A}$	$P(\bar{A}\bar{A}) = \frac{48}{52} \cdot \frac{48}{52} = \boxed{\frac{144}{169}}$

what if you draw 3 Cards with replacement

$P(\text{All aces}) = \frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52} = \boxed{\frac{1}{2197}}$

Apr 4-10:09 AM

A multiple-choice quiz has 4 questions. Each question has 5 choices with only one correct choice.

We are making random guesses.

1) $P(C) = \frac{1}{5}$

2) $P(\overline{C}) = \frac{4}{5}$

3) $P(\text{All Correct guesses}) = \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} = \boxed{\frac{1}{625}}$

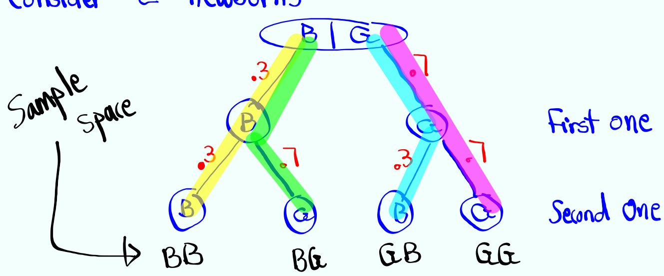
4) $P(\text{All incorrect guesses}) = \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} = \boxed{\frac{256}{625}}$

Apr 4-10:18 AM

Multiplication Rule with Tree diagram.

Suppose $P(\text{Boy}) = .3$ $\quad P(\text{Girl}) = .7$

Consider 2 newborns



$P(BB) = (.3)(.3) = \boxed{.09}$ ✓ $\qquad P(GB) = (.7)(.3) = \boxed{.21}$ ✓

$P(BG) = (.3)(.7) = \boxed{.21}$ ✓ $\qquad P(GG) = (.7)(.7) = \boxed{.49}$

$P(\text{At least 1 boy}) = 1 - P(\text{No boys}) = 1 - .49 = \boxed{.51}$

$P(\text{At least 1 girl}) = 1 - P(\text{No girls}) = 1 - .09 = \boxed{.91}$

$\boxed{\text{SG 12}}$ ✓

Apr 4-10:25 AM

$P(A) = .2$, $P(B) = .5$ A & B are independent events

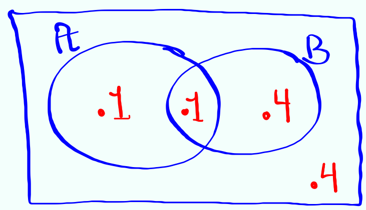
1) $P(\bar{A}) = 1 - .2 = \boxed{.8}$

2) $P(A \text{ and } B) = P(A) \cdot P(B) = (.2)(.5) = \boxed{.1}$

3) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= .2 + .5 - .1 = \boxed{.6}$

4) Make Venn Diagram.



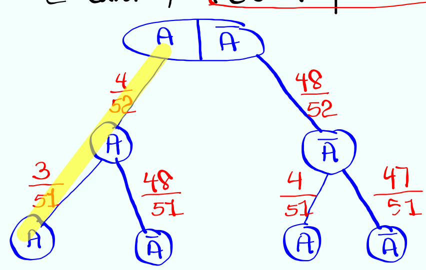
Total = 1

Apr 4-10:35 AM

Full deck of playing Cards

52 Cards 4 Aces

Draw 2 Cards, No replacement



$P(AA) = \frac{4}{52} \cdot \frac{3}{51} = \boxed{\frac{1}{221}}$ $P(\bar{A}\bar{A}) = \frac{48}{52} \cdot \frac{47}{51} = \boxed{\frac{188}{221}}$

$P(\text{at least 1 ace}) = 1 - P(\bar{A}\bar{A}) = 1 - \frac{188}{221} = \boxed{\frac{33}{221}}$

AA
AĀ
ĀA
ĀĀ

$P(\text{at least one not ace}) = 1 - P(AA)$

AA
AĀ
ĀA
ĀĀ

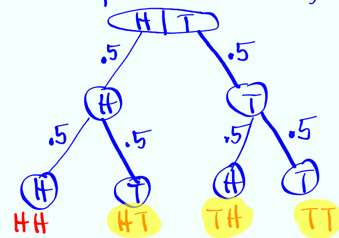
$= 1 - \frac{1}{221}$

$= \boxed{\frac{220}{221}}$

Apr 4-10:40 AM

Consider tossing a fair coin twice,

Draw a Complete Tree Diagram.



$P(\text{at least 1 tail}) = 1 - \underbrace{P(HH)}_{\text{we don't want}}$

Total Prob.

$= 1 - (.5)(.5)$

$= \boxed{.75}$

odds in favor of landing at least 1 tail

P(at least 1) : P(None)

.75 : .25 $\rightarrow$ $\boxed{3:1}$

Apr 4-10:52 AM